COMPARING PARTIAL AND GENERAL EQUILIBRIUM ESTIMATES OF THE WELFARE COST OF INFLATION

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Reserve banks worldwide have been moving towards zero inflation policies. Confusion clouds the welfare cost of maintaining such inflation policies despite the best attempts at clarification. Monetary theory research has shifted from partial to general equilibrium economies. This shift has left the partial equilibrium estimates of the welfare cost of inflation below most of the general equilibrium estimates. Put on a comparable basis, partial equilibrium estimates compare more closely with the general equilibrium estimates. Furthermore, evidence suggests that integration under the money demand function appears applicable in general equilibrium economies. Finally, the estimates depend on the elasticities of money demand and the underlying structural parameters.

I. INTRODUCTION

Estimates of the welfare cost of inflation serve vital functions in research and policy. They help in comparing model economies and in evaluating the policy of sustained inflation. Partial equilibrium estimates confuse these tasks and fall well below newer general equilibrium estimates. Calculations here suggest that lower mean partial equilibrium estimates result because of incomplete accounting of costs, different bases for the calculations, and assumed interest elasticities at the low end of the range. General equilibrium estimates also display a larger variance, which evidence suggests results from a greater diversity in the underlying money demand functions. As Friedman (1956), Bailey (1956), and Eckstein and Leiderman (1992) suggest, trustworthy welfare cost estimates require trustworthy money demand functions.

Inflation imposes a broad array of costs (Dowd, 1992). Baumol's (1952) and Bailey's (1956) "shoe-leather" costs represent resources used in avoiding a sustained inflation tax through alternative exchange technologies. Bailey's (1992) review suggests that these costs provide a lower bound on the total costs of inflation. In partial equilibrium, the utility-based formula measures the real value of the surplus under the money demand curve that the inflation tax eliminates. Lucas (1993, p. 1) states that in general equilibrium, "The thought experiment underlying the formulas is exactly the same as that used in Bailey's (1956) original study"—that is, a determination of the real cost of compensating a consumer for losing utility as a result of being taxed at some rate of inflation.

Cost estimates of inflation tax avoidance give Cagan (1956), Bailey (1956), and Eckstein and Leiderman (1992) a basis on which to evaluate seignorage policy. These estimates provide Fischer (1981) and Lucas (1981) with a platform to debate the scope of monetary theory and supply Cooley and Hansen (1989, 1991, 1992) with a means to study a Friedman and

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Schwartz (1963) type shock on business cycles and tax policy. Gromme (1994) and Black et al. (1993) use such cost estimates to analyze endogenous growth. And as Carlstrom and Gavin (1993) and Braun (1994a) discuss, the cost of zero inflation demands attention as reserve banks move towards such policies (see Dotsey, 1991; Leigh-Pemberton, 1992; Fuhrer and Moore, 1992).

The problem in using the estimates as a standard for analysis is that they differ so much across the literature. A shift from partial to general equilibrium analysis has fragmented the estimates and made comparing them difficult. Consider, for example, estimates of the welfare cost as a percent of GNP resulting from a 10 percent inflation. Partial equilibrium estimates range from 0.22 percent, (Eckstein and Leiderman, 1992) to 0.45 percent (Lucas, 1981). The general equilibrium estimates come in as low as 0.11 percent (Cooley and Hansen, 1989) and as high as 7.15 percent (Marquis and Reffett, 1994).

Seen on a comparable basis, the partial equilibrium estimates in section II depend largely on the assumed interest elasticities of money demand. Further, the methods of partial equilibrium in section III give good cost approximations for some example general equilibrium economies. Variations among the general equilibrium estimates in section IV are partly due to elasticity differences in the underlying money demand functions.

II. PARTIAL EQUILIBRIUM DIFFERENCES

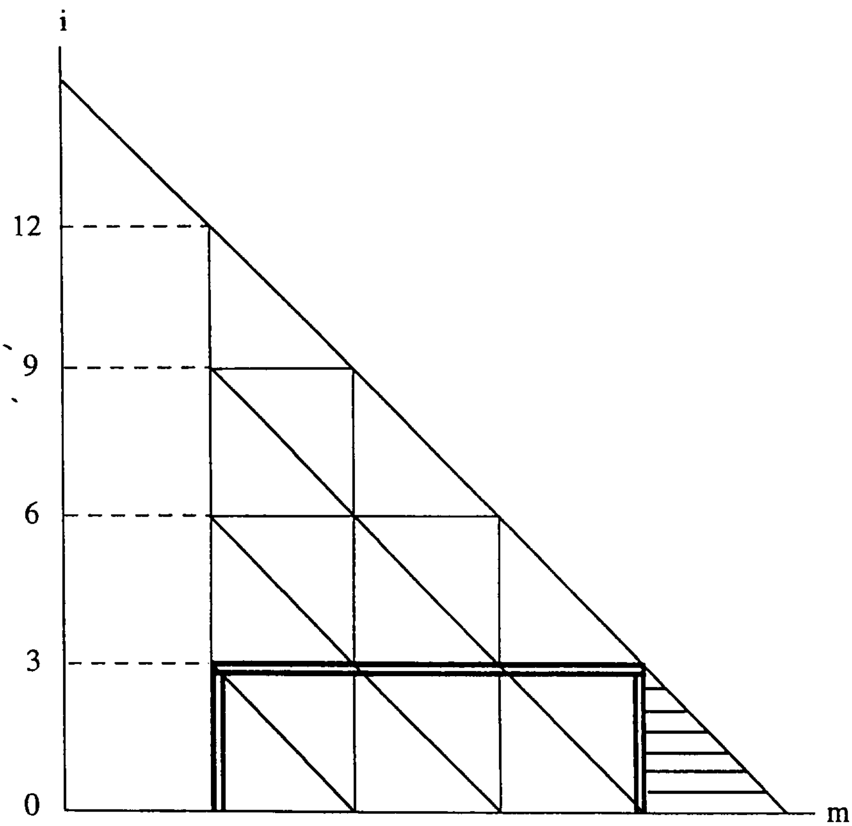
Different bases have led researchers to establish low "priors" for the magnitude of the estimates. The problem of selecting the basis at which welfare costs equal zero, goes back to Friedman's (1953) "Inflationary Gap" article. Friedman describes a 10 percent inflation rate as "a stable price level plus a tax of 10 percent per year on the average amount of cash balances." But does a stable price level already impose a positive or a zero level of taxation? As in Friedman (1969), Bailey (1956) states that the inflation "tax" is zero at a nominal interest rate of zero. This means that the stable price level imposes a positive tax and that the tax makes positive the welfare cost of a stable price level. However, as Tower (1971) emphasizes, Bailey calculates welfare costs as being equal to zero at a stable price level. He then calculates the welfare costs of a zero to 10 percent inflation rate increase as a triangle of lost consumer surplus instead of as a triangle plus the box below it (see figure 1).

Setting the zero-cost basis at the zero inflation rate instead of at the optimal inflation rate would be unimportant if the resulting difference in estimates were negligible. Yet the difference can exceed 50 percent depending on the money demand specification. For a linear money demand, the Tower box in figure 1 represents an amount that is similar to what Bailey mathematically omits. With a 3 percent real interest rate and a zero to 9 percent increase in the inflation rate, this box geometrically equals 6/15 or 41 percent of the lost surplus.

For 1980 M1 data, an approximation of a Cagan money demand function, and a constant semi-interest elasticity of –5, Lucas estimates the welfare loss at 0.45 percent of GNP. To keep the estimate comparable with Bailey's measure, Lucas uses the same cost basis of a zero inflation rate. For correctness, Lucas references Frenkel's (1976) Cagan-based measure that uses the zero nominal interest rate as the zero-cost basis. Thus, the 0.45 percent estimate omits the Tower-type box. For 1989 M1 data, the Cagan money demand function, and a semi-interest elasticity of –5, the Tower-like box is 0.228/0.577 or 39.5 percent of the more inclusive Frenkel measure.

For the central partial equilibrium money demand functions, table 1 shows that omitting the Tower-type box decreases estimates by 38 percent to 51 percent. For the Cagan function, table 1.A re-

FIGURE 1
The Tower Box (Double-Lined) and Frenkel-Triangle (Cross-Lined)



ports the underestimation at 38 to 41 percent. For a constant interest elasticity, table 1.B reports the underestimation at 45 to 51 percent.

The range of the assumed increase in the inflation rate also affects the estimates. Measuring the cost of the 10 percent inflation rate as compared to the *optimum* rather than to a zero inflation rate is a common practice in the general equilibrium estimates. This practice corresponds to adding another "triangle" to the zero-to-10 percent cost estimate. The cross-lined triangle in figure 1 shows this triangle, which Frenkel describes as the welfare loss due to the "non-payment of interest on money." For the constant semi-elasticity function, table 1.A shows that this Frenkel triangle adds approximately 5.6 percent to the cost estimate. For the constant elasticity function, table 1.B shows that the triangle adds from 37 percent to 93 percent to the estimate. The increase is less for the constant semi-interest elastic function than for the constant interest elastic function because of the hyperbolic shape of the constant elasticity function.

Table 1.C shows that the constant semi-elasticity and the constant elasticity estimates can be similar even though they be-

TABLE 1

Comparison of Partial Equilibrium Estimates as a percent of income; 1989 U.S. M1 = 783.7, GNP = 5234, $i = 0.0811$; $i = \pi + \rho$; $\rho \equiv .03$

A. Constant semi-interest elasticity: Cagan: $m = ce^{-\alpha\pi}y$; Frenkel: $m = ce^{-\alpha(\pi+\rho)}y$.[a]

Semi-elasticity α	constant c	π: 0 to 10% Welfare cost: Cagan function (1)	constant c	π: 0 to 10% Welfare cost: Frenkel function (2)	"Tower Box" (3)	Box as % of (2)	π: –.3 to 10% Welfare cost: Frenkel function (4)	"Frenkel Triangle" [(4)-(2)] as % of (4)
1	0.1576	0.00074	0.1624	0.00119	0.00045	37.9	0.00126	5.6
3	0.1745	0.00215	0.1910	0.00351	0.00136	38.7	0.00371	5.6
5	0.1933	0.00349	0.2246	0.00577	0.00228	39.5	0.00609	5.6
7	0.2141	0.00477	0.2642	0.00800	0.00323	40.4	0.00844	5.5
9	0.2372	0.00600	0.3107	0.01022	0.00422	41.3	0.01078	5.4
10	0.2496	0.00660	0.3369	0.01133	0.00473	41.8	0.01194	5.4

B. Constant interest elasticity: $m = ci^{-\eta}y$.[b]

Elasticity η	constant c	0 to 10% Welfare cost: (1)	"Tower Box" (2)	Box as % of (1)	–.3 to 10% Welfare cost (3)	"Frenkel Triangle" [(3)-(1)] as % of (3)
0.10	0.1165	0.00151	0.00068	44.7	0.00206	37
0.20	0.0906	0.00306	0.00139	45.6	0.00443	45
0.30	0.0705	0.00465	0.00216	46.3	0.00724	56
0.40	0.0548	0.00629	0.00297	47.2	0.01074	71
0.50	0.0426	0.00799	0.00383	48.0	0.01537	92
0.60	0.0332	0.00976	0.00478	49.0	0.02200	125
0.70	0.0258	0.01162	0.00528	49.8	0.03264	181
0.80	0.0201	0.01357	0.00687	50.7	0.05338	293

TABLE 1 continued

Comparison of Partial Equilibrium Estimates as a percent of income; 1989 U.S. M1 = 783.7, GNP = 5234, $i = 0.0811$; $i = \pi + \rho$; $\rho \equiv .03$

C. Constant semi-interest elasticities α and interest elasticities η

Welfare costs α: (1)	α	Interst rate: i	conversion $\alpha \cdot i = \eta$	Welfare costs η: η	(2)	Comparison of welfare costs [(2)–(1)]
0.00119	1	0.0811	0.0811	0.0811	0.00122	0.00004
0.00351	3	0.0811	0.2433	0.2433	0.00374	0.00023
0.00577	5	0.0811	0.4055	0.4055	0.00638	0.00061
0.00800	7	0.0811	0.5677	0.5677	0.00918	0.00118
0.01022	9	0.0811	0.7299	0.7299	0.01219	0.00197
0.01133	10	0.0811	0.8110	0.8110	0.01379	0.00246

[a]The formula for column (1) is $w/y \equiv (c/\alpha)[1 - e^{-\alpha(.10)}(1 + \alpha[.10])]$; for column (2) $w/y \equiv (c/\alpha)e^{-\alpha(.03)}[1 + \alpha(.03) - e^{-\alpha(.10)}[1 + \alpha(.13)]]$; for column (3) $c(.03)[e^{-\alpha(.03)} - e^{-\alpha(.13)}]$; for column (4) $(c/\alpha)e^{-\alpha(.0811)}[e^{\alpha(.03)} - (1 + \alpha(.03))]$.

[b]The formula for column (1) is $w/y \equiv c(\eta/[1 - \eta])\,[.13^{1-\eta} - .03^{1-\eta}]$; for column (2) $c(.03)[.03^{-\eta} - .13^{-\eta}]$; for column (3) $w/y \equiv c(\eta/[1-\eta])[.13^{1-\eta}]$.

have differently across the range of interest rates. Excluding the Frenkel triangle, the last column of table 1.C shows that an elasticity conversion with the market interest rate makes the cost estimates nearly equivalent. This explains how estimates from the Cagan function can be low relative to the constant elasticity function. The difference results mainly from the different magnitudes of the Frenkel triangle.

In addition to the contribution of the Tower-box and the Frenkel-triangle, table 1 also shows that the assumed interest elasticity largely determines the magnitude of the estimate. The well-known 0.3 percent estimate fits into the low end of the range in table 1.B. Assuming a –0.25 constant interest elasticity, a monetary base aggregate, and the Friedman (1969) basis, Fischer (1981) calculates a 0.3 percent cost for a 10 percent inflation rate instead of for a zero inflation rate. (The correct estimate with Fischer's assumptions and methodology is 0.17 percent rather than 0.3 percent).[1] Assuming a –0.20 constant interest elasticity, an M1 aggregate, and the Friedman basis, McCallum (1989) approximately reproduces the Fischer convention with a 0.28 percent estimate. In table 1.B, the 0.28 percent number rises

1. Fisher (1981) assumes that the monetary base is 150, GNP = 2600, $i = .12$, the real rate of interest .02, and $m = i^{-.25}yc$. Then

$$c = (150/2600)(.12)^{.25}$$

$$= .03396;\ w/y \equiv \int_{.02}^{.12} (.25)i^{-.25}(.03396)\ di =$$

$$(.03396/3)\ (.12^{.75} - .02^{.75}) = .001706$$

slightly to 0.31 percent as a result of using 1989 data instead of McCallum's 1987 data. Table 1.B shows that taking the measure from the optimum to 10 percent instead of from zero to 10 percent, increases the estimate of 0.31 percent by 45 percent to 0.44 percent. More significantly, however, an increase in the –0.2 constant interest elasticity up to a mid-range of –0.5 more than triples the welfare cost estimate to 1.54 percent.

Table 1 illustrates the factors that have helped make the partial equilibrium estimates low in comparison to the general equilibrium estimates. Omitting the Tower box or the Frenkel triangle or choosing a low interest elasticity knocks down the partial equilibrium estimate. In contrast, the 1.54 percent estimate uses a mid-range constant elasticity, sheds the low "priors," and yields a partial equilibrium estimate more squarely within the general equilibrium range.

III. PARTIAL VERSUS GENERAL EQUILIBRIUM

The longevity of the partial equilibrium estimates depends on whether integrating under the money demand function can yield an accurate estimate in general equilibrium economies. For example, Dotsey and Ireland (1994) report that their partial equilibrium-style estimate yields "only a fraction" of the actual general equilibrium cost (reported in table 2). However, evidence here suggests an integrity of such methods. The broader question instead becomes one of plausibility: what factors determine the cost estimate?

Take, for example, Gillman's (1993) general equilibrium estimate of 2.19 percent. This estimates the cost of a 10 percent, non-optimal inflation rate from the general equilibrium closed form cost function. The interest rate equals 0.133: the 0.10 inflation rate plus the assumed time preference of 0.03 plus a factor of 0.003 that accounts for the discrete time framework. To derive a partial equilibrium-style estimate in the same economy, consider again the Harberger-type formula of table 1 for welfare costs w as a percent of income y. (The cost of a zero to 0.133 interest rate increase can be measured either by the Harberger (1974) measure

$$\int_{.00}^{.13} i \frac{\partial m}{\partial i} di$$

or the Hotelling (1938) measure

$$\int_{m[.00]}^{m[.13]} i \cdot dm,$$

where i denotes the interest rate and m denotes real money demand.) Let η denote the (positively defined) interest elasticity of money demand and write the cost function as

$$w/y = \int_{0}^{.133} \eta m \, di.$$

Consider substituting in from Gillman an approximation of the given interest elasticity (his equation [23]). In particular, dropping the negligible last term gives the elasticity as

$$\eta = [(i/Aw)/(1 - [i/Aw])] + i/(1 + i),$$

where $Aw = 0.54$ denotes the calibrated cost of exchange credit. Writing the money demand (Gillman's equation [29]) as $m = (1 - i/Aw)c_1$ with c_1 denoting the cash good, the cost formula becomes

$$w/c_1 = \int_{0}^{.133} \{(i/.54) + (i/[1 + i])(1 - [i/.54])\} \, di.$$

Finally, multiplying through by c_1/y, calibrated from his equations (4) and (26) as (0.2774/0.2828), gives the partial equilib-

TABLE 2
General Equilibrium Estimates

	Inflation Experiment	Money stock	Interest elasticity, or *semi-elasticity*	Welfare cost estimate*
Money in the utility function				
Eckstein and Leiderman (1992)	0 to 10%	n.a.	0.24	0.85–1.93%
Lucas (Section 2,1993)	optimum to 10%	M1	*7*	1.64%
Cash-in-Advance				
Cooley and Hansen (1989)	optimum to 10%	base	n.a.	0.11%
	optimum to 10%	M1	n.a.	0.39%
Cooley and Hansen (1991)	optimum to 10%	M1	n.a.	0.36%
Gillman (1993)	-2.9 to 10%	M1	0.43	2.19%
Ireland (1994)	optimum to 10%	n.a.	n.a.	0.62%
McCallum and Goodfriend exchange				
Den Haan (1990)	0 to 5%	n.a.	n.a.	4.60%
Black, Macklem, and Poloz (1993)	0 to 10%	M1	0.31	3.04–3.14%
Lucas (Section 3,1993)	optimum to 10%	M1	0.5	1.50%
(Section 5, 1993)	optimum to 10%	M1	0.13	1.00%
Braun (1994a)	optimum to 4%	base	0.55	0.95%
Dotsey and Ireland (1994)	0 to 10%	base	*2.73*	0.92%
		M1	*5.95*	1.73%
Growth Theory context				
Gromme (1993)	optimum to 8.5%	n.a.	n.a.	0.03%
Black, Macklem, and Poloz (1993)	0 to 10%	M1	0.31	4.82–5.06%
Marquis and Reffett (1994)	optimum to 10%	M1	*0.04*	7.15%
Dotsey and Ireland (1994)	0 to 10%	base	*2.73*	0.20%
	0 to 10%	M1	*5.95*	0.92%

rium style estimate of $w/y = 2.28$ percent. This compares closely in magnitude to the exact 2.19 percent estimate.

Note that integrating the money demand function does not necessarily imply holding constant the marginal utility of income. It remains unclear how to hold this fixed in general equilibrium, as one might attempt in order to simulate Marshall's partial equilibrium description. For example, Gillman's (1994a) cash good c_1 exactly equals the inverse of the real marginal utility of a dollar. Holding this marginal utility constant means fixing the consumption of the cash good. Yet the basic experiment is to test the consumer's response to inflation. For one approach, however, consider again the interest elasticity in Gillman (1993). It breaks down into the interest elasticity of approximately the inverse of the money velocity in the first term, $(i/Aw)(1 - i/[Aw])$, plus the interest elasticity of the cash good in the other two terms. Dropping the last two terms, holding the marginal utility of a dollar constant in some sense, and recalculating the partial equilibrium style estimate gives a 29 percent lower estimate of $w/y = 1.61$ percent.

Lucas (1993) provides another example of how integration under the general equilibrium money demand function yields an estimate that compares well with the general equilibrium estimate. As table 2 indicates, he provides four estimates from three different general equilibrium economies. From the money-in-the-utility function, Sidrauski (1967)-type economy, Lucas (section 2) first calculates a Taylor-type approximation of $w/y \approx (0.89)i^2$, or 1.57 percent for $i = 0.133$. With a 0.5 elasticity of substitution between money and goods, Lucas derives a second more exact cost estimate for this economy of $w/y = (0.45)i^{0.5}$, or 1.64 percent for $i = 0.133$. Compare these estimates to the Harberger-type triangle by integrating the money demand function. Lucas provides this as $m = i^{-1/(1+\xi)}[\delta/(1+\delta)]^{-1/(1+\xi)}y$. Making the assumption that $\xi = 1$, for an elasticity of substitution of 0.5, Lucas calibrates that $\delta = 0.998$. This gives

$$w/y = \int_0^{i_0} \eta m \, di = (0.45)i^{0.5}, \text{ or } 1.64 \text{ percent.}$$

It equals Lucas's exact estimate. The result strikingly indicates an applicability of the partial equilibrium methods.

Equivalently, assume as in table 1.B that $m = ci^{-0.5}y$. Solve for c as $c = i^{0.5}/v_0$, where v_0 is the given period velocity. This gives an alternative formula for the partial equilibrium integration:

$$w/y = \int_0^{i_0} \eta m \, di = (\eta/([1 - \eta])(i/v_0).$$

In Lucas's Sidrauski-style economy (Sidrauski, 1967), the interest elasticity is constant at −0.5 and velocity equals $[(.998/.001)i]^{0.5}$. Making the substitutions yields the same formula of $w/y = (0.45)i^{0.5}$ and the same estimate of 1.64 percent. With this alternative partial equilibrium-type approach, just three numbers determine the estimate: the interest elasticity, the interest rate, and the velocity.

For practical purposes, Lucas's other two economies and the corresponding estimates show the limits of using partial equilibrium methods. In section 3, Lucas specifies a McCallum and Goodfriend-type (McCallum and Goodfriend, 1987) exchange economy and approximates the general equilibrium welfare costs by $w/y \approx (0.41)i^{0.5}$. The cost estimate equals 1.50 percent for $i = 0.133$. This compares closely to the section 2 estimates of 1.57 percent and 1.64 percent. But an integration approach faces hurdles here. The underlying money demand function, as derived from Lucas's equations (3.8, 3.10, 3.11, 3.13), equals $m = (.2866)i^{-0.5}y^{0.5}$ and includes a 0.5 income elasticity. As a result, one must include a value for income in order to calculate by the Harberger triangle method.

Second, consider Lucas's final estimate from an extended McCallum and Goodfriend (1987) economy in section 5. This results from a complex general equilibrium closed form function of the interest rate. While the analysis here does not present this formula, one can approximate the cost estimate of about 1.00 percent for an interest rate of 0.133 from Lucas's table 3. The underlying money demand function, which can be computed from the equilibrium solution, has a unitary income elasticity. But the money demand function remains quite complex. Computing the Harberger triangle may be no easier than computing the general equilibrium closed form cost function.

Partial equilibrium methods can offer simple, accurate formulas for the general equilibrium economy. Yet they offer no guarantee of a less complicated approximation than do the general equilibrium methods. Offering an alternative to the partial equilibrium methods, Lucas's (1993) paper emphasizes that general equilibrium approximations can give simple formulas for the estimates. These formulas depend on the interest rate and on the underlying structural parameters: from preferences in Sidrauski-type economies or from the exchange technology in McCallum and Goodfriend-type economies.

The general equilibrium approximation advantageously reveals the layer beneath the partial equilibrium elasticities. For example, the Taylor approximation of the inflation cost in Gillman's (1987) cash-in-advance economy depends on the calibrated cost of exchange credit, $Aw = .54$, and on the log-utility preference for leisure, $\alpha = 2.27$:

$$w/y \approx [(1/[1+\alpha])(1+[1/Aw]$$

$$-[1/(1+\alpha)])][i^2/2]/.2818.$$

For $i = 0.133$, this estimate equals 2.44 percent as compared to the exact estimate of 2.19 percent. Besides a simpler formula than the closed form function for the exact estimate, the approximation reveals the likely comparative statics of the structural parameters, just as Bailey (1956) and Lucas (1981) put the focus on the effects of the behavioral parameters. This clarifies testable hypotheses—for example that the cost estimate will trend upwards because the cost of exchange credit trends downwards.

IV. GENERAL EQUILIBRIUM DIFFERENCES

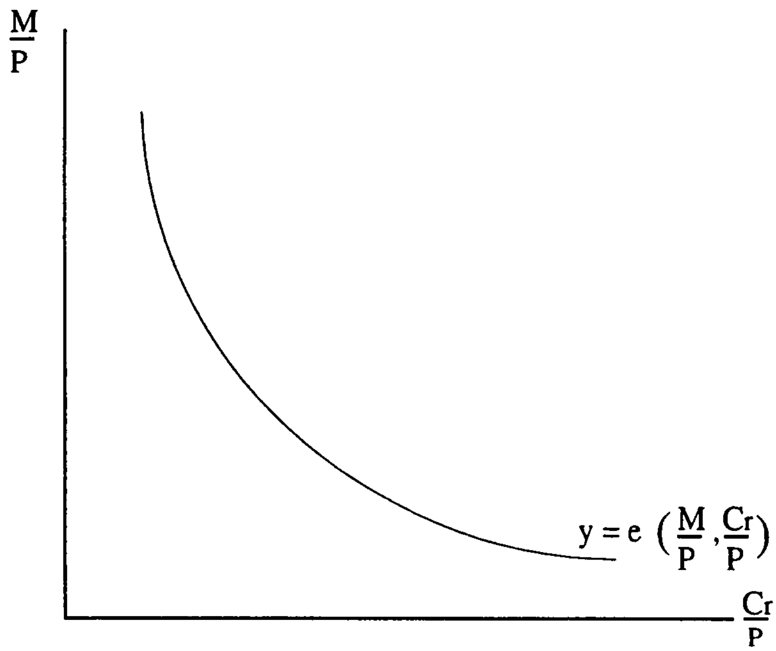
Differences in calibrated structural parameters and their effects on the economies cause differences amongst the general equilibrium estimates given in table 2. Figure 2 (from Gillman, 1994b) shows a way to view the effect of the specification of the exchange parameters in terms of the implied interest and income elasticities of money demand. It shows the combination of real money M/P and real credit Cr/P along an isoquant that represents a given amount of exchange. The level of exchange is produced by the function $e(M/P,Cr/P)$ and equals the level of real output y in equilibrium: $e(M/P,Cr/P)=y$. With a unitary income velocity of money, the parameter specifications would require the function e to be homogeneous of degree one in M/P.

The curvature of the isoquant in figure 2 depends inversely on the elasticity of substitution between money and credit. In Gillman (1993), the interest elasticity exactly equals the elasticity of substitution between cash and credit plus a factor for changes in the marginal utility of income (see Gillman, 1994a). With a high interest elasticity, the curvature is slight, the decrease in utility from tax distortions is large, and the welfare cost of inflation is high.

For example, the Cooley and Hansen (1989) estimate of 0.39 percent (quarterly data) results when the consumer can avoid inflation only with substitution from goods to leisure. With no exchange credit, the money demand is relatively inelastic. Reproducing the leisure-only channel, Gillman (1993) estimates a comparable cost at 0.58 percent. Adding a cash-to-costly-credit channel and maintaining an approximate unitary income elasticity, Gillman's estimate rises from 0.58 percent to 2.19 percent as the interest elasticity rises from 0.11 to 0.43.

Black et al. (1993) offer similar evidence. Including a cash-to-costly-credit channel and a unitary income elasticity, they find a cost estimate of 3.04 percent and an interest elasticity of 0.31 for a 10 percent inflation rate. Including endogenous economic growth in the analysis, they report a higher estimate of the welfare cost of inflation than for a comparable

FIGURE 2
An Isoquant for Exchange



economy found in Gromme that lacks the cash-to-costly-credit channel. The Bailey-type interest elasticity link also helps explain the magnitude of the estimates reported by Lucas (1993), Braun (1994a), Dotsey and Ireland (1994), and Eckstein and Leiderman (1992).

V. CONCLUSION

The nature of the welfare cost of inflation supplies evidence for monetary theory. This helps us "to work 'toward isolating numerical constants of monetary behavior" (see Lucas, 1988, p. 137; Friedman, 1956). The paper finds reasons why the 0.3 percent standard of partial equilibrium estimates is low and supplies evidence on why the method of integrating under the money demand function remains valid. The wide variance of the general equilibrium estimates apparently results from various specifications of the exchange technology and the related structural parameters. Neither partial nor general equilibrium contradicts a central concept of Bailey (1956)—that is, the higher the interest elasticity of money demand within a given economy, the more the substitution to costly exchange alternatives and the higher the welfare cost of inflation. Assuming a comparable basis, a mid-range interest elasticity, and a 10 percent non-optimal inflation rate, a conservative estimate range is 0.85 percent to 3 percent for the different economies reported here. In terms of U.S. GNP for 1994, a cost of 0.85 percent translates into a loss of $58 billion.

Research could identify further the linkage between the structural parameters, the elasticity features of money demand, and the welfare cost estimates. Focusing on exchange credit markets' technology and distortions and on the ability to avoid the inflation tax also may refine cost estimates (see Gillman, 1994a; Lucas, 1993; Ireland, 1995). Different optimum quantities of money also affect the cost estimates. For example, the optimum occurs at a zero inflation rate in Gillman (1995) when accounting for menu costs and at a positive inflation rate in Braun (1994b) when incorporating a Ramsey tax framework. Dynamically, an advancing technology of credit production implies increasingly less expensive credit alternatives, more substitutes to cash, and a more interest elastic cash demand. This suggests that the base welfare cost of a given inflation rate will trend upwards and that sustained inflation will become an increasingly less attractive policy.

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