Optimizing the Capital Rationing Decision with Uncertain Returns

Liuqing Mai*

College of Business Administration
University of Missouri - St. Louis
One University Blvd
Saint Louis, MO 63121-440
E-mail: mailiu@umsl.edu
Phone: (314) 516-5567

Haitao Li

College of Business Administration
University of Missouri - St. Louis
One University Blvd
Saint Louis, MO 63121-440
E-mail: lihait@umsl.edu
Phone: (314) 516-5890

Abstract

In this paper, we develop a new optimization model for capital rationing with uncertain project returns. Our model maximizes the probability of meeting a pre-defined target return by selecting a feasible set of projects subject to budget constraints in multiple time periods. We employ a mixed-integer nonlinear algorithm recently developed in the optimization field to solve the resulting non-convex optimization problem to optimality. Our model and solution methods are tested and validated through a comprehensive computational experiment. Several managerial insights are obtained on the impact of available budget and target return on the optimal solutions. Notably, we have found that increasing target return may not necessarily result in increase of optimal total expected return of the selected projects. Our model and solution method offers a computationally tractable approach to quantify the tradeoff between project returned and risk, and optimize capital rationing decisions under uncertainty.

Keywords**:** Capital rationing, uncertain returns, nonlinear optimization

1. Introduction

Capital rationing aims to allocate limited resources for multiple risky investment opportunities or projects (cf. (Fremgen 1973), (Gitman and Forrester 1977)). It is part of the general capital budgeting process (cf. (Pinches 1982), (Mukherjee and Henderson 1987)), and plays an important role in aligning firm's resources with its strategic goals and operational needs. Factors impacting capital rationing decisions include the project return, level of project cash flow, and resource/budget limits per time period. In a typical capital rationing problem, given a set of project opportunities with expected return and resource requirements, one needs to make a feasible selection of projects, so that the available resource limits are not exceeded and some performance measure of the selected projects is optimized.

The discounted cash flow method to appraise an investment project for its net-present-value (NPV) is a well-established approach in capital budgeting. The NPV of an investment is computed by the present value of future incomes subtracting the investment costs. Under certainty, a firm should select the project opportunities with positive NPVs. This approach assumes unlimited budget or resources, thus does not explicitly serve the need of capital rationing.

In the seminal work of (Weingartner 1963), a linear programming (LP) model was developed to maximize the net present value (NPV) of a project, which is computed as a function of time-dependent cash flow and a discount rate. Their model is *deterministic* in nature in that all input data concerning project return, cash flow and budget limits are assumed to be constants. While such deterministic optimization approach for capital rationing has been routine for decades since the early 1980s (Gitman and Mercurio 1982), its main drawback is that the uncertain financial impact of project opportunities has not been explicitly dealt with.

Uncertainties are ubiquitous in financial investment and may impact capital budgeting decisions (cf. (Schall and Sundem 1980), (Simerly and Li 2000)). (Miller 1992) proposed an uncertainty framework with three general types of uncertainty: general uncertainties affecting all companies (e.g., inflation, interest, exchange rate), industry-specific uncertainties impacting some specific industries (e.g., input market, output market, competitive), and organization specific uncertainties which affect only a particular organization (e.g, production, labor). A different risk framework has been proposed by (Collier and Berry 2002), which includes four domains of risks: financial, operational, political and personal. The existence of uncertainty and risk makes the NPV of a project to be random variable with dispersions, rather than a constant. Under such circumstance, the firm concerns not only the return of investment, but also the risks involved.

Risk analysis and assessment in financial investment has been intensively studied. Well-known approaches include the risk-adjusted discount rate (Fama 1977), the certainty equivalent (Sick 1986), and the well-known capital asset pricing model (CAPM, (Brick and Weaver 1984)). While these methodologies primarily focus on determining appropriate discount rates and risk premiums, they do not directly address the capital rationing decision. Recent growing financial risks and volatile economic environment call for more sophisticated methods for capital rationing under uncertainty (cf. (Verbeeten 2006), (Singh, Jain et al. 2012)).

In this paper, we develop a new model and solution approach that directly optimizes capital rationing decisions under uncertainty, and in particular, uncertain return of project opportunities. Our model optimizes the probability of achieving certain threshold on the total expected return, subject to budget constraints over the planning horizon. With the assumption that a project return is a random variable and follows independent normal distribution, we show that the objective function has a closed form, but is nonlinear. Solving such nonlinear optimization problem to optimality is often computationally challenging. We employ a computational algorithm developed in mathematical programming, called the polyhedral branch-and-cut algorithm (Tawarmalani and Sahinidis 2005), to obtain optimal solution for the problem at hand.

Our work makes the following contributions to the research field. Through optimizing the probability of reaching the threshold of total expected return, our model offers an explicit and meaningful approach to address the impact of uncertain return. With the use of an exact optimization algorithm and comprehensive computational experiment, we are able to obtain useful managerial insights about optimal capital budgeting solutions under variation of problem input data. Our model and solution approach provide an effective and efficient decision-support tool for practitioners to optimally balance project return and risk.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the problem setting, and formally presents the model formulation, solution method and variation/extension to the basic model. Section 4 presents computational experiment and insights obtained. Section 5 draws conclusions and discusses future research opportunities.

2. Literature Review

Given the vast research literature in capital budgeting, our review will focus on methodologies that directly consider uncertainty in optimizing capital rationing decisions.

A classical approach for balancing the return and risk of investment opportunities is the well-known mean-variance model (Markowitz 1952), which optimizes the tradeoff between the total expected portfolio return and risk measured by total portfolio variance.

The mean-variance model relies on a quadratic programming formulation, with the quadratic term representing the total variance/covariance of the selected candidate entities (Cornuejols and Tutuncu 2007). An efficient frontier of best return-risk pairs can be obtained through sensitivity analysis. The main strength of the mean-variance model is its ability to capture the covariance of uncertain returns, and optimally diversify the portfolio. To address capital rationing decision, details about the cash flow of project opportunities and cost of capital must be considered.

In the review of (Verbeeten 2006), real option and game theory approaches are considered as being sophisticated capital budgeting practices (SCBP). Real option is a well-known approach to value financial investment under uncertainty (Miller and Waller 2003). A key feature of the real option approach is its ability to model sequential decisions/options dynamically over time. For one project opportunity, the decision-maker has the options to accept, to postpone, to expand or to abandon a project at different decision points of the planning horizon ((Trigeorgis 1993), (Burgess and Busby 1992)). The value of the investment varies with decision/option chosen, as well as uncertainties involved. The main benefit is to achieve flexibility in capital budgeting decisions that are adaptive with the realized project cash flows. The classical real option approach, however, lacks means to handle project selection decisions with budget constraints. (Meier, Christofides et al. 2001) developed an integrated framework combining the strengths of real option and mathematical programming for capital rationing under uncertainty.

The game theoretic approach addresses capital budgeting decisions under competition. Here the payoff/return of an investment opportunity not only depends on characteristics of the opportunity itself, but also the other player's decision in certain game setting. Under such circumstance, there is usually incentive for a firm to invest early to avoid sacrificing later after a competitor preempts the investment opportunity (cf. (Smit and Ankum 1993), (Zhu and Weyant 2003)). Other research integrates real option and game theoretic methods to enhance the performance of either approach alone (Smit 2003).

Another line of research applies the stochastic programming (SP, (Birge and Louveaux 2011)) methodologies to optimize capital rationing decision under uncertainty. Two SP paradigms have been implemented. The first one is known as a two-stage SP model with recourse, which makes a "here-and-now" first-stage decision, then a second-stage "recourse" decision for each possible scenario of uncertain parameter realization. Among others, (Kira, Kusy et al. 2000) modeled shortage and surplus of funds as the recourse decision to handle uncertain budget constraints. The second paradigm is called chance constraint programming, which directly works with probabilistic constraints. Notably, (Sarper 1993) studied uniformly distributed cash flows and transform the model into a deterministic equivalent. (Beraldi, Bruni et al. 2012) developed a more general chance constraint programming model with joint probability constraints and proposed a branch-and-bound exact method.

3. Optimization Model and Methods

This section starts with a formal description of the setting and assumptions of capital rationing problem addressed in this paper. We then present its mathematical formulation and the solution method.

3.1. Problem Description

Consider a set I of project investment opportunities. Each project $i \in I$ has a projected return $\tilde{r}_i$, which is uncertain at the point when the decision is made. For each period t of the time horizon $t = 1, \dots, T$, there is a budget limit of K_t that cannot be exceeded. Each project $i \in I$ has a constant cost of capital c_{it} in period t. Given a threshold π, the decision maker needs to select a portfolio of projects to maximize the probability of the total expected portfolio return being no less than π. In the basic version of the problem, we assume that the project return $\tilde{r}_i$ for $i \in I$ is independently distributed following a normal distribution $\mathcal{N}(\bar{r}_i, \sigma_i^2)$, with mean $\bar{r}_i$ and variance σ_i^2.

3.2. Model Formulation

Define binary decision variables x_i to model the project selection decision, such that $x_i = 1$ if project i is selected, and $x_i = 0$ otherwise. Then the objective function can be written as:

$$Max\ Prob[\textstyle\sum_{i \in I} \tilde{r}_i x_i \geq \pi] \tag{1}$$

The main constraint of the model is the multi-knapsack type constraint, which ensures that total cost of capital in each time period t does not exceed the available budget K_t.

$$\textstyle\sum_{i \in I} c_{it} x_i \leq K_t \qquad \forall t = 1, \dots, T \tag{2}$$

Next we show that with the assumption of independent normal distribution for project return, the objective function (1) can be expressed as a closed-form nonlinear expression. The standard deviation $\mathcal{V}$) of the total return of selected projects can be computed as:

$$\mathcal{V} = (\textstyle\sum_{i \in I} \sigma_i^2 x_i)^{1/2} \tag{3}$$

We then have:

$$Prob[\textstyle\sum_{i \in I} \tilde{r}_i x_i \geq \pi] = Prob[\mathcal{N}(0,1) \geq (\pi - \sum_{i \in I} \bar{r}_i x_i)/\mathcal{V}] \tag{4}$$

Therefore, the objective function (1) is equivalent to the minimizing the z-value in standard normal distribution:

$$Min\ (\pi - \textstyle\sum_{i \in I} \bar{r}_i x_i)/\mathcal{V} \tag{5}$$

Optimal capital rationing (project selection) decision can be obtained by solving the formulation (5) plus (2) and (3), which is an integer nonlinear program.

We comment the relationship of the above model with some existing models in the literature. It is a direct extension of a multi-knapsack problem or the classical Lorie-Savage problem for capital budgeting (Lorie and Savage 1955) by considering uncertain project return and minimizing a nonlinear objective function. It also extends the stochastic knapsack problem in the optimization field (cf. (Steinberg and Parks 1979), (Morton and Wood 1998)) by having multiple knapsack-type constraints.

3.3. Solution Method

Due to the non-convex form of the objective function (5), it will be difficult for typical branch-and-bound based mixed-integer nonlinear programming (MINLP) methods to work properly and prove optimality (Floudas 1995), because these methods often rely on the convexity assumption of the model. The existence of multi-knapsack constraints also make the dynamic algorithm (DP) proposed by (Morton and Wood 1998) less efficient.

In this paper, we employ a global optimization algorithm, called polyhedral convex relaxation branch-and-cut, developed by (Tawarmalani and Sahinidis 2004) to solve our integer nonlinear model for capital rationing. This algorithm successively generates cutting planes to linearize and approximate the original feasible region. Bounds are obtained through such outer approximation of the feasible region. Comparing with other heuristic global optimization methods, its main advantage is the ability to prove optimality, so that the decision-maker is able to know whether a solution is optimal, and if not, how good it is. The polyhedral relaxation branch-and-cut algorithm is readily available in the Baron Solver of GAMS (Sahinidis 2013).

4. Computational Results

The purpose of our computational experiment is two-fold. Firstly, to examine the behavior of optimal capital rationing solution in different problem spaces characterized by key input parameters such as number of projects, available budget capacity level and target return. Secondly, to understand how our solution performs comparing with the deterministic solution based on point estimates (PE) of project return, which maximizes the total expected return alone. A numerical example is presented next to obtain several observations about optimization approach. Then we show computational results of a comprehensive experiment.

4.1. A Numerical Example

A firm is considering a set of 10 investment project opportunities with random return following independent normal distribution $\mathcal{N}(\mu, \sigma^2)$. Each project's mean return μ, variance σ^2 and budget requirement c_{it} over five years are shown in Table 1. The available budget each year is \$38M, \$31M, \$33M, \$31M, \$15M. The firm is attempting to achieve a total target expected return of \$50M, and would like to find a feasible selection of projects that maximizes the probability of meeting the target, without violating the available budgets.

Table 1. Return and budget requirement of 10 project opportunities.

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
μ (\$M)	7	12	14	13	12	5	16	11	4	7
σ^2	15	20	15	10	8	20	8	15	20	25
c_{i1} (\$M)	5	7	11	9	8	4	12	10	3	6
c_{i2} (\$M)	2	5	10	8	8	3	10	6	4	5
c_{i3} (\$M)	4	8	8	6	7	4	10	8	5	5
c_{i4} (\$M)	3	7	9	7	7	4	9	7	4	4
c_{i5} (\$M)	2	3	3	4	3	2	3	4	3	2

Table 2 compares the optimal solution found by our solution approach with the one found by deterministic PE approach. The deterministic solution obtains the highest possible total expected return of \$53M, but may end up having a large variance of return, thus sub-optimal probability to meet the target return. In this case, our optimal solution achieves the highest probability (67.08%) of meeting the target return, with the same total expected return of \$53M, but significantly less variance.

Table 2. Comparison of optimal and deterministic solutions.

	Total Expected Return (\$M)	Total Variance of Return	Prob to Meet Target Return (%)
Optimal Solution	53	46	67.08
Deterministic Solution	53	73	63.73

To better understand the behavior of optimal capital rationing solution, we perform sensitivity analysis to examine the impact of input parameters on optimal solutions. In our analysis, we focus on the effect of target revenue and budget availability. We let the ratio of available budget over the base case vary evenly between [0.82, 1.2] with an increment of 0.02. We also vary the target return in the same way. This generates 20 by 20, or 400, instances with different combinations budget level and target return.

The impact of such change on the optimal probability of meeting the target return is shown in Figure 1. When the available budget increases, the firm is able to select more projects, if possible, to increase the probability of meeting the target return, as evident in Figure 1. Note that such relationship does not appear to be smooth, but piece-wise linear, due to the combinatorial nature of the addressed problem. That is, optimal probability may stay the same if a budget increase is not significant enough. In addition, the slopes of different linear pieces appear to be different, indicating that for the same amount of budget increase, the increase in optimal probability may be different, depending where the starting budget level is on the curve. Our optimization approach will assist the firm to understand precisely the benefit of each budget increase. Observation 1 follows.

When the target return increases, the optimal probability of meeting it decreases with the same available budget. Such relationship appears to be smooth because the target return resides in the objective function, z-value, and the probability density function of normal distribution is continuous. We have Observation 2 below.

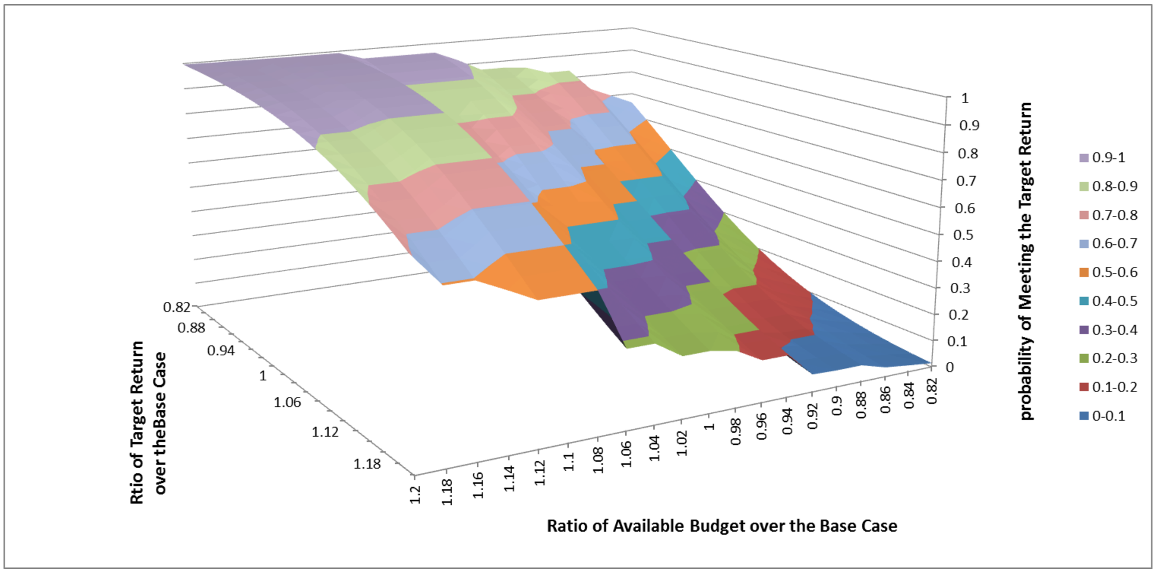


Figure 1. Optimal probability of meeting target return when available budget and target return vary.

OBSERVATION 1. Budget increase will increase the optimal probability of meeting the target return in a piece-wise linear fashion.

OBSERVATION 2. Increase of target return will reduce the optimal probability of meeting the target return in a continuous fashion.

The optimal expected total return shows a similar piece-wise linear increasing trend as the optimal probability when the available budget increases, as shown in Figure 2. The change of target return alone, however, does not appear to impact the optimal expected total return. We may state Observations 3 and 4 below.

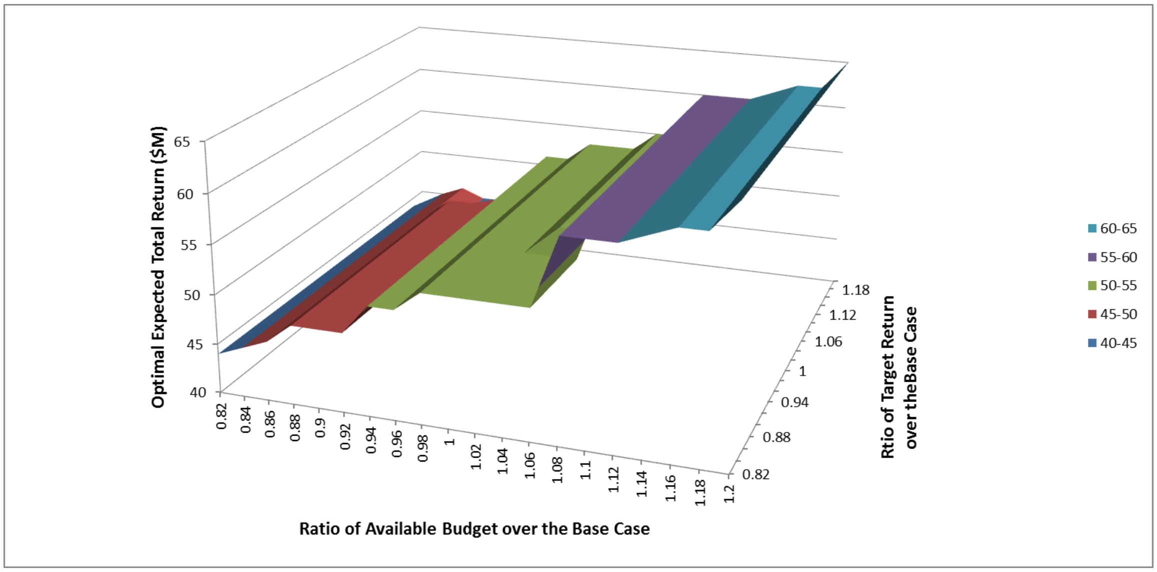


Figure 2. Optimal expected total return when available budget and target return vary.

OBSERVATION 3. Budget increase will increase optimal expected total return in a piece-wise linear fashion.

OBSERVATION 4. Change of target return will not have significant impact on optimal expected total return.

We next examine in Figure 3 the relative performance of our optimization solution comparing with the deterministic solution when both available budget and target return vary. It is observed that the advantage of optimal solution diminishes when the available budget is either very ample or very scarce. This is because: (i) when the available budget is very ample, the optimization problem becomes trivial with the budget constraints being non-binding; (ii) when the available budget is tight, the optimal probability of meeting target return will be small enough so that there is really not much difference between an optimal and deterministic solution. More improvement of optimal solution can be achieved when there is medium level of budget available. We state Observation 3 below.

OBSERVATION 5. The benefit of optimal solution over deterministic solution diminishes when the available budget is either very ample or every tight.

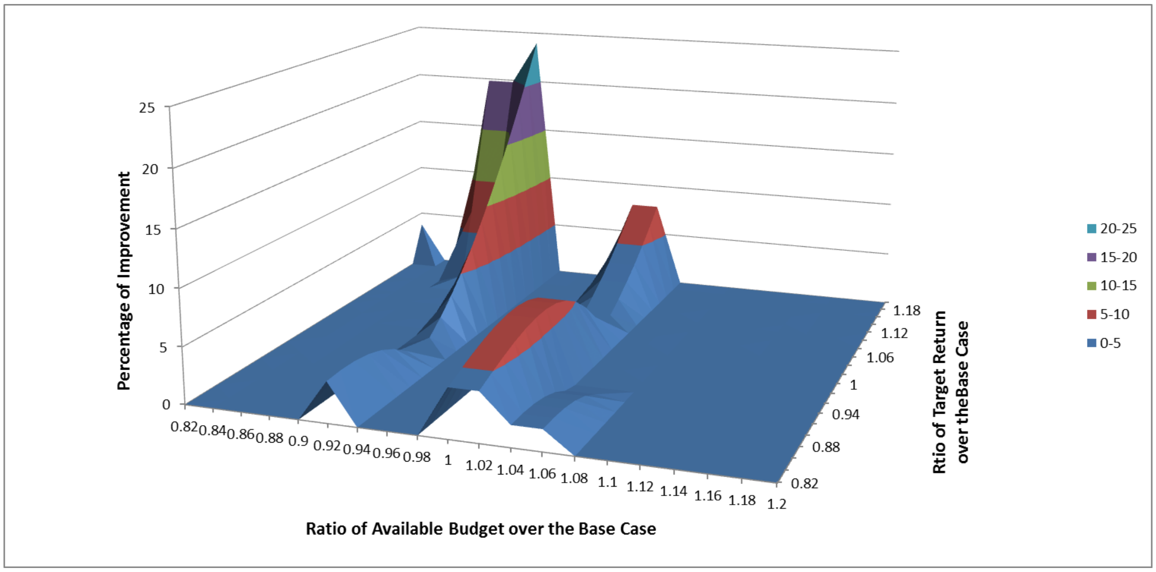


Figure 3. Percentage of improvement of optimal solution over the deterministic solution.

4.2. Experiment Results

We design and perform a comprehensive experiment to verify and validate the observations obtained in the previous section. The parameters controlled in our experiment are presented in Table 3. We consider three different problem sizes with 10, 25 and 50 project opportunities. The available budget of a time period is controlled by a ratio over the total budget requirement in the period. For instance, with the ratio being 0.5 and the total budget requirement of all project opportunities being \$100M in Period 1, the available budget for Period 1 is set to be \$100M × 0.5 = \$50M. We vary the ratio between the interval [0.3, 0.48] with a step size of 0.02. The target return is controlled in a similar way by a ratio over the total expected return of all project opportunities, and we let it vary in the interval [0.3, 0.7] with a step size of 0.1. For each combination of project size, budget availability and target return, 10 replicates are generated, which gives a total of $3 \times 10 \times 5 \times 10 = 1{,}500$ instances. For each instance, we randomly generate the mean return, variance of return, and budget requirement per time period of each project opportunity, assuming they all follow uniform distribution as shown in Table 3. Each instance is solved by our optimization model and the deterministic approach.

Table 3. Parameters and their values in the experiment.

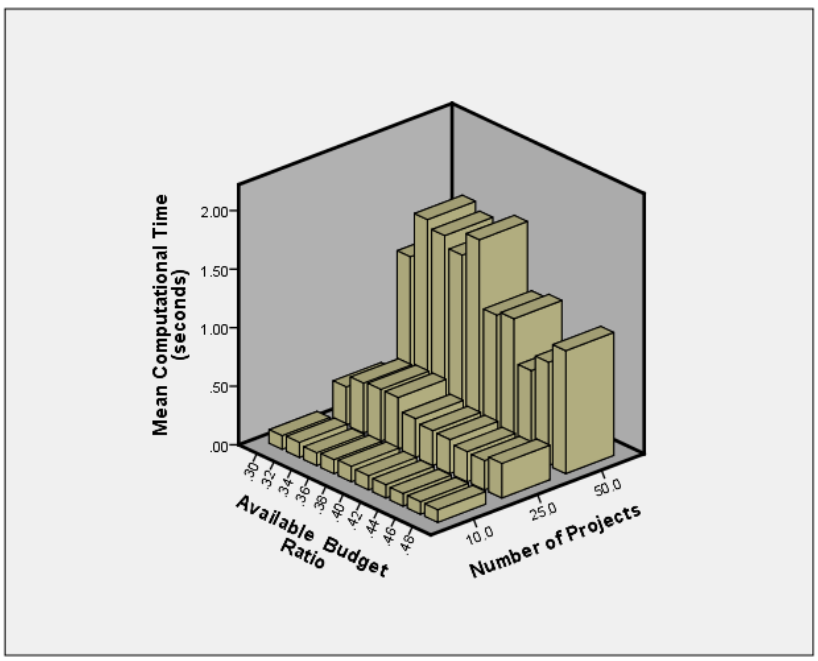
Parameters	Nature	Number of Levels/ Type of Distribution	Values
Num. of projects (N)	Controlled	3	{10, 25, 50}
Budget availability (B)	Controlled	10	[0.3, 0.48] with a step size of 0.02
Target return (T)	Controlled	5	[0.3, 0.7] with a step size of 0.1
Replicate	Controlled	10	
Mean of project return	Random	Uniform	$U[5, 10]$

Var of project return	Random	Uniform	$U[10, 25]$
Budget requirement	Random	Uniform	$U[1, 12]$

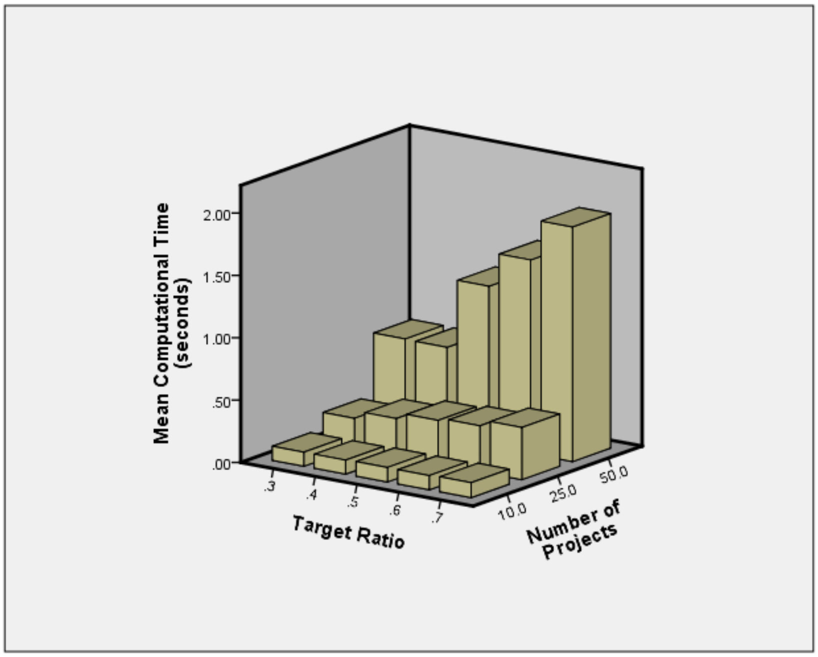
Figure 4 shows computational time for the polyhedral branch-and-cut algorithm to solve the 1,500 instances. The algorithm is very efficient: it takes fractions of a second to solve 10-project instances and 1.23 seconds to solve the 50-project instances on average. Problem size is the main factor of computational effort, that is, computational time increases as the number of projects increases. It appears in Figure 4(a) that the computational time does vary with the available budget, and a tighter budget may require more computational time. Hypothesis 1 follows. From Figure 4(b) we may observe that the computational time increases when the target return increases. We may state Hypothesis 2 below.

HYPOTHESIS 1. The average computational time increases for instances when the available budget becomes tight.

HYPOTHESIS 2. The average computational time increases for instances with higher target returns.



(a) When available budget varies.



(b) When target return varies.

Figure 4. Computational time for the test instances.

Corresponding with Observations 1 and 2, we state Hypothesis 3 regarding the optimal probability of meeting the target return. Hypothesis 4 can be stated as indicated by Observations 3 and 4.

We use linear regression analysis to test Hypothesis 1 ~ 3, with number of projects N, budget availability B and target return T as independent variables. Table 4 reports the regression results. The numbers in parentheses are the standard errors of the coefficient estimates.

In Model 1 (M1), the dependent variable is the computational time in seconds for solving the instances. All the coefficient estimates are significant with $\alpha = 0.01$. The negative sign of B indicates that the computational time increases when the available budget decreases. The positive sign of T verifies that increasing the target return will increase the computational time. Thus Hypothesis 1 and 2 are supported. Note that the adjusted R-square is only about 16%, suggesting that majority of the variation in computational time cannot be explained by the regression model. This is mainly due to the combinatorial nature of the problem, where even a small change in input data may significantly impact the computational effort to solve it.

In Model 2 (M2), the dependent variable is the optimal probability of meeting the target return. All the coefficient estimates are statistically significant. The positive sign of

B indicates that increasing available budget will increase the optimal probability; while the negative sign of *T* corroborates that increasing target return will decrease the optimal probability. Therefore, Hypothesis 3 is supported. Note that here more than 80% of variations in optimal probability of meeting target return can be explained.

In Model 3 (M3), the dependent variable is the optimal total expected return of selected projects. The coefficient of *T* is insignificant, thus the target return does not have statistically significant effect on the optimal total expected return. The positive sign and large value of the coefficient of *B* indicate positive relationship between the available budget and optimal total expected return, which supports Hypothesis 4. More impressively, near 98% variation in total expected return can be explained.

HYPOTHESIS 3. The optimal probability of meeting the target return will decrease when the target return increases; it will increase when the available budget increases.

HYPOTHESIS 4. The optimal total expected return will increase when the available budget increases; it will not be affected by the target value.

Table 4. Linear regression results.

Dependent Variable	Constant	*N*	*B*	*T*	Adj. R^2
Computational time (M1)	-0.237* (0.237)	0.029 (0.002)	-1.57 (0.523)	1.19 (0.212)	16.1%
Optimal probability of meeting target return (M2)	0.87 (0.039)	0.004 (0.000)	2.22 (0.087)	-2.655 (0.035)	81.4%
Total exp. return (M3)	-167.244 (3.539)	6.947 (0.027)	389.511 (7.79)	1.030* (3.164)	97.8%

Figure 5 shows the percentage of improvement of the optimal solutions over the deterministic solutions. It clearly indicates that the advantage of our optimization approach is not evenly distributed in different problem space, which corroborates the finding of Observation 5.

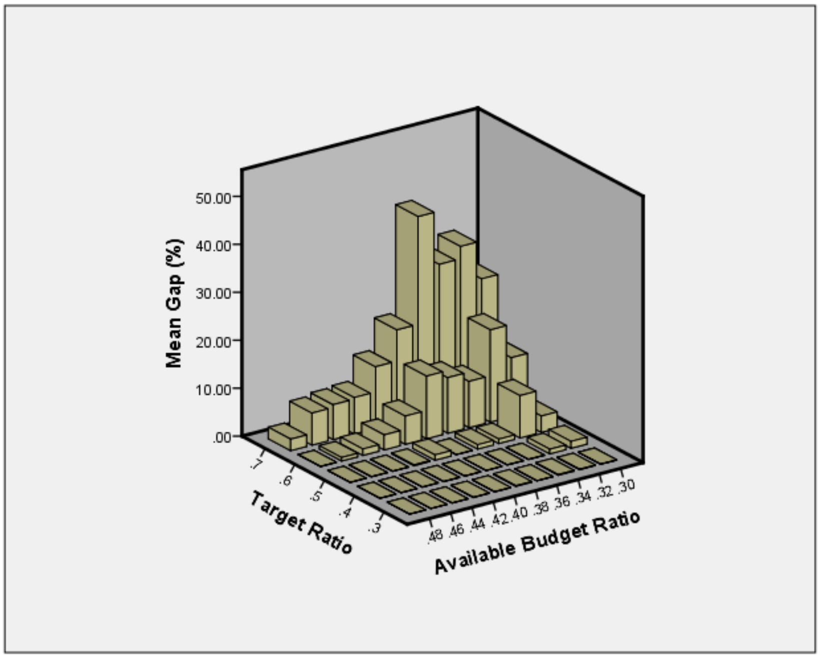


Figure 4. Percentage of improvement when both target ratio and available budget ration vary.

5. Conclusion and Future Research

In this paper, we have developed a new model to optimize capital rationing decisions with uncertain project returns. Our model minimizes the probability of meeting a target total return subject to budget constraints in each period of the planning horizon. Under the assumption that project returns follow independent normal distributions, it results in solving a mixed-integer linear program (MINLP). We apply the polyhedral branch-and-cut algorithm of to solve the MINLP. Our model optimizes the tradeoff between project return and risk/variation by formulating a probability-type objective function. The advantage is its capability of assisting a firm to better assess the impact of risk on capital rationing decisions.

We have designed and performed a comprehensive computational experiment to examine the behavior of optimal solutions, and compare its performance with that of a deterministic approach which minimizes the total expected return. Our optimization model and the proposed solution method based on the polyhedral branch-and-cut algorithm have shown computational efficiency: it spends less than two seconds on average for solving instances with 50 projects. We have also obtained several insights. Firstly, as budget increases, the optimal probability of meeting target revenue will increase in a piece-wise linear fashion; and as target return increases, the optimal probability will decrease continuously, but nonlinear fashion. Our model and the corresponding sensitivity analysis offer a rigorous way to precisely quantify the impact of available budget and target value on capital rationing. Secondly, while it is not surprising that the optimal total project return will increase when the available budget increases, we have found that increasing target return will not necessarily increase the optimal total expected return, which is somewhat counterintuitive. When a firm's target return is set to be high, the decision-maker may be often under the pressure to favor projects having higher mean return. Our optimization model and solution suggest that when the target return increases, one should still optimally balance the project and return, thus the total expected return of the resulting selected project portfolio may not increase. Thirdly, through comparison of our optimal capital rationing solution with the deterministic solution, we have quantified the advantage of our approach in different problem space characterized by problem input data. We find that a firm may gain most benefit when the available budget is neither too high nor too low.

Our work has opened an avenue of research opportunities. Future study is needed to develop models and solution methods for the case with general probability distributions of project return. There, the challenge is that no closed-form objective function may exist, thus the MINLP methods employed in this paper will not be applicable. It calls for the development of some methodologies combining the optimization and simulation techniques. Another direction is to extend the current static one-period model in a multi-period dynamic setting, where the firm has more flexibility in their capital rationing decision over time. For example, the company may shrink or expand, delay or fasten a project; it may also accept new project opportunities arrived in the funnel and abandon an existing project.

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